

Transforming Post-Secondary Education in Mathematics

Report of a Meeting

University of Texas at Austin
June 20–22, 2014



TPSEMath

Transforming Post-Secondary Education in Mathematics

About TPSE Math

Transforming Post-Secondary Education in Mathematics, abbreviated as TPSE Math, is an informal effort aiming to bring constructive change in mathematics education to community colleges, 4-year colleges and research universities.

Through meetings, conference calls and social media, TPSE Math is soliciting input from a wide range of stakeholders including students, faculty members, university administrators, and employers. Among the topics being addressed are:

- How well post-secondary curricula fit the needs of mathematics majors, other STEM majors, non-STEM majors, and potential employers
- How institutions can remove barriers and open pathways into and through the mathematical sciences curriculum
- How departments can improve teaching effectiveness while managing fiscal pressures on the traditional economic model
- How local experiments to enrich the undergraduate experience can be shared and scaled up
- How graduate students, most of whom find careers outside R1 universities, can gain teaching, mentoring, and communication skills

Coordinating the effort are Phillip Griffiths, Institute for Advanced Study (Convener); Eric Friedlander, University of Southern California; S. James Gates, Jr., University of Maryland; Mark Green, University of California at Los Angeles; Tara Holm, Cornell University; Uri Treisman, University of Texas at Austin.

The first public discussion of TPSE Math took place at the Joint Mathematics Meetings in Baltimore in January 2014. This report summarizes discussions that took place at a larger meeting hosted by the University of Texas at Austin June 20-22, 2014.

The first phase of TPSE Math (2013–2014) has been supported by grants from Carnegie Corporation of New York and the Alfred P. Sloan Foundation. The TPSE Math group is now planning additional regional meetings to continue the conversation, and seeking funding to carry proposed reforms beyond the experimental stage.



Introduction

As a research discipline unto itself, a complement to other STEM fields, and an underpinning of career preparation and an informed citizenry, mathematics has evolved rapidly in recent years and grown in importance.

With this evolution have come new challenges: introducing more undergraduates to the relevance, beauty, and uses of mathematics; expanding the traditional math curriculum to embrace interdisciplinary fields and prepare students for the workplace; educating students who are poorly prepared for post-secondary math; adapting to new teaching technologies; and preparing graduate students to be educators as well as researchers.

As noted in the recent PCAST report *Engage to Excel*,¹ mathematics is seen as the number one barrier to college completion at a time when the nation needs many more mathematics majors, non-majors with more extensive and deeper mathematics preparation, and STEM majors who are better prepared for the mathematically intensive aspects of life sciences, social sciences, engineering, information technology, business, and security.

Other barriers to effective mathematics education are institutional, cultural, and political. Among them are high levels of student debt, reduced public funding, pressure from students' parents for small classes and intensive mentoring, and the skepticism of state legislators who question the value and expense of college education. What used to be regarded as a public good is now criticized as a private benefit.

Largely lost in many debates is the fact that having a world-class STEM workforce will benefit the entire nation. This realization is not new: As the National Research Council concluded more than two decades ago, "Prosperity in today's global economy depends on scientific and technological strength, which in turn is built on the foundation of mathematics education."² Since then, that foundation has changed little.

2 | In response to the PCAST report, the National Research Council's *The Mathematical Sciences in 2025*,³ and a growing sense of urgency throughout the mathematics community, a group of mathematicians gathered in 2013 to better inform themselves about the current dialogue around these issues, and to work with others toward collaborative solutions.⁴ This group, under the rubric Transforming Post-Secondary Education in Mathematics, or TPSE Math, found that many individual faculty and departments were experimenting with innovative approaches, but experienced little communication among themselves or with the administrations that also seek reforms. Hence TPSE Math adopted the goal of disseminating what is known and what is needed, and reaching out to the mathematics community and its many stakeholders.

The first phase of TPSE Math (2013-2014) has been supported by grants from Carnegie Corporation of New York and the Alfred P. Sloan Foundation. Its first public discussion was convened at the 2014 Joint Mathematics Meetings in Baltimore as an attempt to distill the issues and offer an initial framework of questions.⁵ These included:

- How well post-secondary curricula fit the needs of mathematics majors, other STEM majors, non-STEM majors, and potential employers
- How institutions can remove barriers and open pathways into and through the mathematical sciences curriculum
- How departments can improve teaching effectiveness while managing fiscal pressures on the traditional economic model
- How local experiments to enrich the undergraduate experience can be shared and scaled up
- How graduate students, most of whom find careers outside R1 universities, can gain teaching, mentoring, and communication skills

Several months later, TPSE Math organized a more extensive and comprehensive meeting at the University of Texas at Austin, attended by 67 mathematicians and other stakeholders from 44 institutions (see Appendix 1).

General Recommendations from the Austin Meeting

Discussions in both plenary and breakout sessions at the University of Texas were organized under five issues. These issues are listed below, along with a general recommendation meant to suggest the central themes and outcomes of the discussion. This section is followed by more detailed Findings and Recommendations submitted by each of five breakout groups.

Curriculum Reform

Mathematical science departments must re-evaluate their curricula to provide experiences appropriate to the skill levels and motivations of diverse students, including mathematics majors, other STEM majors, and non-STEM majors.

Mathematical sciences departments should build networks with mathematics-intensive disciplines, and include those partners in curriculum review efforts. Meaningful collaboration is needed among all post-secondary institutions, including research universities, comprehensive four-year institutions, liberal arts colleges, and community colleges.

Removing Barriers and Opening Pathways

Institutions must create multiple pathways into and through the mathematical sciences curriculum, offering a mix of choices for math majors as well as other STEM and non-STEM majors and communicating the wide range of math-related career options.

They must also better align gateway courses with both high school exit courses and major programs of study; develop better tools for recruiting students to math; strengthen the role and status of non-tenured faculty; and raise incentives to reward the practice of education as well as research.

Teaching and the Economic Model of Mathematics Departments

Departments should enhance the status of teaching/pedagogy in the local culture, rewarding both tenure-track and non-tenure-track faculty in collaborative efforts at curriculum and teaching reform.

To manage fiscal pressures, departments should work with administrations to reinvest savings from new instructional technologies to improve teaching outcomes. They should also use reward structures for faculty that are based less on individual action and more on collective responsibility.

Enriching the Undergraduate Experience

The undergraduate experience should be enriched by exposure to statistics, modeling, data science, and computation, as well as more experiential learning, effective use of technology, and career advice. Change must be generated by grass-roots innovations and shared through collaborative efforts at the national level.

Within universities, partnerships with other disciplines are essential, as are multi-lateral discussions with stakeholders of diverse types, sizes, and regions.

Enhancing the Graduate Training Experience

Because most graduate students find careers outside R1 universities, their training should include development of teaching, mentoring, and communication skills, and internships in the private or public sectors.

In addition, advanced graduate students should have charge of their own undergraduate classroom and gain expertise in the uses of mathematics in other STEM disciplines.

The self-selected breakout groups that addressed these issues in Austin were asked to submit Findings and Recommendations. These are summarized below, along with several related issues.

Group 1 / Curriculum Reform

The group that discussed curriculum reform addressed issues spanning the breadth of programs for the four major populations of post-secondary mathematics students: mathematics majors, majors in other STEM fields, non-STEM majors, and high school graduates with insufficient preparation in mathematics.

General finding

The role of the mathematical sciences has broadened and deepened rapidly, with math-intensive specialties spreading through other disciplines. This provides a need and an opportunity to re-imagine the curriculum in a form that is better adapted to the requirements of students, faculty, departments, institutions, and the mathematical community. Many such efforts are happening, but they generally occur in local settings and have not been brought to scale or linked with other efforts.

General recommendation

Mathematical science departments must **re-evaluate their curricula** so that they provide experiences appropriate to the skill levels and motivations of the diverse range of students, including mathematics majors, other STEM majors, and non-STEM majors. In meeting this challenge, it is critical to **strengthen communication** between mathematics faculty and faculty in unrelated majors, downstream departments, and two- and four-year institutions.

Specific findings

1. As documented in the “Math 2025” report, the role of the mathematical sciences throughout society has expanded considerably. New areas have opened up, and existing connections between fields have deepened. At the same time, the mathematics required for informed citizenry and for jobs throughout the workplace has grown more complex. These changes demand transformation in the curricula of universities, four-year colleges, and two-year colleges.
2. Many fields, in addition to the physical sciences and engineering—for example, the biosciences, areas of the social and behavioral sciences and business, and the emerging field of data science—have become math-intensive and are presenting new demands on those who design and teach undergraduate curricula. Many other careers now require some mathematics or statistics. There is an increasing need for institutions to expand capacity to meet these needs.
3. Broader societal changes are placing significant pressures on the economics of higher education in many states. Increasing enrollments, rising student debt, and waning support in some states are leading to intense cost pressures in many institutions. These pressures on the economic model of math departments require new ways to deploy resources while enhancing effectiveness.
4. In light of the many new options for instruction and for design of instructional materials, the mathematical sciences community is challenged to revisit course content, delivery methods, and instructional materials. Goals are to develop better educational assessment tools, more consistent use of these tools, and more sharing of what is learned with other institutions.
5. Some institutions report that more entering students now arrive underprepared in mathematics than previously. Changes are needed to match entry-level mathematics instruction with the preparation level of high school graduates under new K-12 state standards.

6. Changes are also needed in the education and in-service training of those who teach mathematics in K-12. Most K-8 teachers are not required to have mathematical training, yet many of the nation's math problems develop in grades 5-8, when American students lag behind those of many other countries.
7. The traditional mathematics curriculum is a barrier for many students hoping for STEM-related careers. And yet the PCAST report, *Engage to Excel*, describes a severe shortage of STEM majors and calls for one million more STEM graduates over 10 years.
8. Fortunately, promising work is underway in many regions and at the national level (see Appendix 2). The best of these projects should be scaled up and coordinated.

Recommendations for curriculum initiatives

1. Mathematical sciences faculty should accelerate curriculum innovation efforts for students in mathematics majors, for students in other mathematics-intensive majors, and for students in majors that are not mathematics intensive. For all of these categories of students, courses should demonstrate ways to connect the mathematics studied to students' intended majors. Departments should provide multiple options within the major, broader experiences, opportunities to learn in depth, and more interesting ways to deliver instruction and engage students.
2. Mathematical sciences departments should strive to build networks with partner disciplines that are mathematics-intensive and include those partners in curriculum review efforts. Meaningful collaboration is needed among faculty from different kinds of post-secondary institutions, including research universities, liberal arts colleges, comprehensive four-year institutions, and community colleges.
3. Participation of the professional societies is needed to provide frameworks and guidance for collaborative work, mechanisms for sharing what is developed, and mechanisms for scaling up the best outcomes of this work.
4. Make available a web-based resource with descriptions of model programs and data describing their effectiveness. This resource will require a process for identifying and using moderators.
5. Provide professional development opportunities that familiarize faculty and adjuncts with ongoing reforms, new instructional materials, options in delivery methods, and opportunities for student learning beyond the classroom.
6. Develop external funding to support faculty time for developing new courses and modify existing ones, evaluate these courses, work with faculty in other institutions, and participate in regional and national meetings.
7. Support professional development for faculty, possibly through new sources such as the NSF mathematical sciences research institutes.

Ten goals for curriculum initiatives

The curriculum group offered a series of goals that might be expected to give rise to and to support the recommended curriculum initiatives:

1. Offer students a broad experience that is connected to the expanding role of the mathematical sciences and incorporates a variety of modes of mathematical thinking, such as deductive reasoning,

- 6 | formal manipulation, modeling and simulation, algorithmic thinking, and probabilistic and statistical reasoning. This experience should grow out of the strengths and inner coherence of the mathematical sciences, and should, when appropriate, further stimulate new mathematics through the uses of the mathematical sciences.
2. Align the curriculum with the skills and understandings students need for a broad range of 21st-century careers.
 3. Provide for majors a deep experience in the mathematical sciences—for example, a research experience, an open-ended project, an internship, or consulting.
 4. Work closely with partner disciplines in designing courses, possibly in the context of an institution-wide convening of stakeholders. This dialogue should also encourage experiments with partner disciplines in collaborative teaching.
 5. Provide a broad palette of teaching methods—accompanied by assessment tools—to determine the effectiveness of various curriculum designs. Engage with the development of online/adaptive education and develop new curricular materials in a variety of media.
 6. Create multiple pathways into and through the mathematical sciences curriculum. Offer, when possible, a mix of choices for math majors. Communicate a wide range of career options as outcomes for students.
 7. Value, respect, and reward those who are involved in educational innovations that advance the educational goals of the department.
 8. Create a community resource (or expand existing resources) for those developing educational innovations and broadening methods of teaching. Enhance communication across institutions.
 9. Provide professional development opportunities to faculty, adjuncts, and graduate students to facilitate implementation of new courses and teaching methods.
 10. Scale up successful programs and share data regarding program effectiveness.

Group 2 / Removing Barriers and Opening Pathways

A central premise of this group was that many features of post-secondary mathematics education create or serve as barriers to student progress. The group sought to better understand these features, and to suggest ways of removing barriers and replacing them with new pathways.

The group discussed barriers of many forms, which may arise

- when high school graduates are ill-prepared for post-secondary “gateway” courses, such as college algebra or calculus;
- when college students avoid mathematics because they believe they cannot understand it;
- when neither students nor faculty understand the many careers available to math and other STEM majors;
- when some part-time faculty are expected to handle the heaviest teaching loads with poor pay, no benefits, and little respect;
- when legislators blame faculty for poor student performance; and

- when partner institutions, such as community colleges and state universities, do not communicate well about their mutual needs and expectations.

The group reported that there is “enormous work going on” by those who would open pathways and gateways, led by institutions, individual faculty, state task forces, professional associations, and many others. At the same time, they found that faculty members are hesitant to make substantial changes in practice amid so many alternatives whose effectiveness is poorly understood.

The group discussed the “dramatic changes in the democratization of higher education,” including “massive income-based differences in how well students are served.” They cited data indicating that of people whose incomes are in the top quartile, 82% get a bachelor’s by age 24; of those whose incomes are in the bottom quartile, only 8% get a bachelor’s by age 24.

General finding

Mathematics is seen by many as the #1 barrier to college completion. High school graduates are ill-prepared for post-secondary “gateway” courses; universities do not communicate well with community colleges, four-year colleges, and smaller public universities; gateway courses are poorly aligned with major academic courses.

General recommendation

To address these findings, the mathematics community must support a major national effort to lower barriers to mathematical advancement and thinking; align post-secondary efforts with those of K-12; develop better recruitment tools to attract students to mathematics; and demonstrate to legislatures and the public the value of a stronger mathematical enterprise.

Specific findings

1. **Increasing numbers of adjunct faculty:** Some two-thirds of post-secondary faculty are part-time adjuncts who typically earn \$2500-3000 per course and have neither benefits nor a tenure track. This creates two- and three-tiered structures, which damage morale and divide the community. Community colleges in particular hire adjuncts to lower costs.
2. **Legislators’ negative view of universities:** Legislatures tend to blame university faculty and administrators for failures of the educational system, including its high tuition rates, high salaries for star faculty, and low employment prospects for many graduates.
3. **The advantage of state-level task forces:** One participant took part in a state-level task force in Ohio, which met with 36 mathematics department chairs. He reported anxiety about poorly understood events and political forces that were shaping their budgets. Some anxiety was lifted when knowledgeable people joined the discussion.
4. **Confusion over credit transfers:** At the University of Texas at Austin, 64% of bachelor’s students transfer in 12 units or more, with many of them underprepared in mathematics. This leaves the university the task of offering ever-expanding developmental courses. The mismatch between preparation at community colleges and standards at universities requires new mechanisms of communication.
5. **Gateway courses are insufficiently aligned with major academic courses.** Gateway courses must be adjusted as high schools respond to college readiness standards.

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6. **How to scale local innovations.** Scaling can be detrimental unless it respects the diversity of higher education. There is no shortage of new practices, but the challenge is to implement them at the regional or national level with proper standards and controls.
 7. **Mathematicians lack good tools for recruiting students.** Few high school or college students appreciate the power and beauty of mathematics. The mathematics community needs better tools to convey the richness of mathematics, its relevance to careers in many fields, and what jobs their alumni actually take.
 8. **The mathematics community lacks good structures for “surfacing” and amplifying consensus,** which is needed to work on these problems. It needs tools for mobilizing people and legitimizing work in areas when there is wide consensus.
 9. **The mathematics community does little public outreach about its mission.** It lacks effective communication to inform the public about what mathematicians do. There are messages from many organizations, but no convincing aggregate voice.

Specific recommendations

1. **Develop better tools for mobilizing the mathematics community.** Because the decision-making function of the math community is so fragmented and individualized, coordinated action is rare even when there is a clear consensus for action.
2. **Align gateway courses with major programs of study,** including the big enrolment majors and re-envisioned minors (e.g., all the sciences, nursing, journalism). Clarity from the field as a whole will help local decision making.
3. **Align post-secondary courses with high school exit courses.** The transition to post-secondary courses is more likely to be successful if both sides coordinate.
4. **Develop better tools for recruiting students to math.** This includes communicating the power and beauty of today’s mathematics. For example, many engineering students choose engineering because they like math, but have little idea of the many pathways to a profession in mathematics.
5. **Strengthen the roles of non-tenure faculty.** There must be respect for adjunct and part-time instructors from institutions and professional associations to enhance their contributions. Also needed are appropriate honoring and reward rituals, opportunities for advancement, quality evaluations, and incentives for innovation.
6. **Review the role of college algebra as the default college-level course.** Multiple entry-level pathways can provide rigorous instruction for those who do not intend programs that require calculus. Focusing college algebra on those students who intend a calculus-based major can deepen the quality of instruction for those students.
7. **The mathematics community should listen more closely to its various publics,** which include parents, legislators, employers, and other STEM fields. The community has been ineffective in reaching out to stakeholders, building a platform for consensus, testing consensus, and acting on it in a coherent way.
8. **Build a community of departments across regions and types of institution.** Departments gain power and perspective through networking with others. Different perspectives lead to better solutions, and each has much to learn from those who got an early start.

9. **Tenure should reward the practice of education as well as the practice of research.** Tenure should be associated with not only “rights” (security, pay, freedom to do research) but also “responsibilities” (mentoring, networking, communicating with stakeholders, collaborating on teaching practices).
10. **Build on the capacity of departments to respond to incentives.** Math departments are usually adept at meeting important targets—such as ensuring that XX% of freshmen succeed in gateway math courses—and devising strategies to do so. Reward units for faculty and student success, with the support of central budgeting.

Group 3 /

Technology, Teaching, and the Economic Model

This group was unique in addressing not one issue but three: technology, teaching, and the economic model of mathematics departments. These issues are linked in important ways—especially in affecting how mathematics departments function.

General finding

Groups across the nation are experimenting with innovative teaching technologies; efforts to move away from a two-tiered structure of teaching; and new ways to hold down costs while increasing the quality of instruction.

General recommendation

Departments and institutions should develop reward structures for faculty that are based less on individual action and more on collective responsibility. Because fiscal pressures are so severe, departments should work with administrations to reinvest instructional savings in departmental capacity.

Technology

Many individuals and groups are developing technologies designed to offload procedural aspects of skill development. A potential benefit of teaching technologies is that they might allow faculty to use classroom time more productively for teaching students to think effectively.

Technology findings

1. In recent years, the use of new technologies in classrooms across the disciplines has risen dramatically, and the technologies are improving steadily. The primary goals of these experiments are to improve teaching outcomes and lower costs.
2. Mathematics faculty are increasingly involved in these experimental efforts. The sophistication of the technology is likely to increase rapidly, raising its potential to have a positive impact.
3. While there has been more hype than substance to date on the uses of technology, interesting and successful examples are emerging, especially in lower-division courses. These examples suggest technology can be a tool that improves teaching outcomes in a cost-effective manner.
4. As a result, technology is changing the modes, expectations, and content of teaching in mathematics.

10 | **Technology recommendations**

1. Mathematics faculty should take the lead in bringing new teaching technologies to the classroom. If they do not take the lead, others will.
2. Departments should partner with other departments, educators, and mathematicians across the institution to incorporate current thinking on course designs, including those that use new technologies.
3. Collect data on the outcomes of using technology in teaching; rigorously evaluate the results; use the information gathered and data available from the institution for continuous improvement in classroom instruction; and carefully track costs.
4. Teach those involved in instruction the use of technologies that enable undergraduates to model phenomena and to ask and answer quantitative questions that arise across disciplines.
5. Promote, publicize, and distribute research on technology-driven innovations.

Group members agreed that mathematics faculty can and must join in a trend that has already begun to bring change to their discipline. As one noted, “Technology is coming anyway. It’s better to embrace it so we will be driving the change.”

Teaching

Traditionally, the mathematics enterprise, especially at R1 universities, assigns a lower status to teaching than to research—a habit encouraged by its tradition of prizes for research excellence and incentive systems in which promotions, tenure, and compensation depend more on research results than on teaching excellence.

Teaching findings

1. In part because of fiscal pressures, departments are creating two categories of faculty: traditional tenure-track faculty, and non-tenured faculty focused on teaching and pedagogy with less job security and with teaching responsibilities primarily at the lower-division level.
2. The tenure-track faculty generally benefit from more favorable salary and reward structures. This development has created the perception and reality of a two-class system of faculty.

Teaching recommendations

1. Work to integrate teaching/pedagogy faculty into the life and culture of the department.
2. Affirm and honor the different tracks pursued by faculty and recognize the value that all faculty bring to the department.
3. All faculty, including tenure-track faculty and the teaching/pedagogy faculty, should engage collaboratively in undergraduate curriculum and teaching reform.
4. Encourage and enable faculty in both tracks to be aware of and incorporate the results of teaching research.
5. Collaborate with other departments at the institution, both in STEM and other disciplines that face similar concerns, to jointly develop undergraduate education reforms.

6. Explore, import, and adapt practices in other disciplines that recognize contributions to the teaching and learning mission.
7. Build collaborative networks with math faculty at neighboring institutions, especially those at community colleges, to better align curricular offerings and reform efforts at the lower division level.

Economic Impact

Economic findings

1. Universities and math departments are under severe economic pressures.
2. These pressures are forcing departments to find new ways to meet their responsibilities.
3. To date, the primary tactic to cut costs has been to hire part-time and adjunct faculty as teachers.
4. The fiscal pressures are unlikely to subside for many, if not most, departments.
5. University administrations are eager to find innovative means of instruction that not only hold down the growth in costs, but also improve results.

Economic recommendations

1. Develop departmental and institutional reward structures that are not based primarily on individual performance, but that recognize collective responsibility and achievement. Incentives might include FTEs, support staff, space allocation, and return of overhead from grants.
2. A structure that recognizes collective responsibility should also recognize that faculty move through different “phases” in careers when there may be more or less time for teaching, research, administrative, or inter-departmental activities. A year spent co-designing a mathematical biology course, for example, might include the expectation of a temporary slowdown in research activity. Actively pursue new teaching and learning strategies that can improve outcomes as well as lower costs.
3. Work with university administrations to develop a system whereby a substantial portion of any instructional savings are reinvested in building departmental capacity.
4. Recognize that if mathematicians do not take the initiative to enumerate *both* the costs and benefits of meeting institutional goals, some other agent or force from outside the mathematics community will define, measure, and probably minimize them.

As noted in the introduction, such goals as promoting a sense of collective responsibility and building an incentive structure rewarding that responsibility are especially challenging for the mathematics community. One reason is the community’s highly diversified and independent structure. This raises the importance of networking and partnerships in promoting reforms.

Group 4 / Enriching the Undergraduate Experience

The Undergraduate Experience breakout group addressed a broad question: Are faculty willing and able to adopt new practices, including co-design and co-teaching, that promote the relevance and uses of mathematics?

General finding

Current undergraduate experiences offer students little understanding of the uses and relevance of mathematics; at the same time, a more quantitatively literate workforce is needed for employment in every sector.

General recommendation

Two broad strategies are needed to drive transformation: Grass-roots efforts driven by local communities, and structural redesign mediated by a large, centralized organization, with support from professional societies and universities. Keys to both are (1) ability to scale up and evaluate innovations, and (2) efforts to build long-term relationships with stakeholder people, departments, and other institutions.

Specific Findings

1. There is a national need for a more quantitatively literate workforce.
2. An increasingly diverse student body reflects the increasing diversity of the nation.
3. A variety of stakeholders depend on mathematical learning, including K-12, community colleges, government, business, and industry.
4. Mathematics has very high rates of drop / fail / withdrawal.⁶
5. The “industrial model” of mathematics instruction prevails, continuing the long-standing lecture/recitation format. This model allows for quick scale-up to larger numbers of students, but may not be the most effective way to teach for understanding and retention.
6. Revenue surpluses from mathematics instruction are central to budget models in most post-secondary institutions, dissuading university administrators from moving to a less efficient but more effective model in times of constrained budgets.
7. Both budget models are more costly than MOOCs, whose role is still uncertain.
8. Few partnerships have been developed between key stakeholders, especially departments, colleges and universities, K-12 programs, and the private sector.
9. These stakeholders share levels of concern and anxiety, but little motivation to learn from one another.
10. Local innovations are seldom adopted beyond their place of origin, and their effectiveness is insufficiently assessed.

The group spent considerable time on the question of scaling up local innovations. It found that “examples of fabulous local innovation abound,” but concluded that these innovations are not yet scaled up to the level desirable for systemic change. It called for rubrics that may be available nationally and applied locally; measures that help assess status and progress; and aids to manage change that has national authority while emanating from the community.

Recommendations

The group agreed on the need to make learning outcomes explicit, and asked what 21st century students need for success in an ever-changing world. It saw at least two means of driving transformation:

1. *Grassroots* efforts driven by local communities, slowly propagating organic change;
2. *Structural change and redesign* mediated by a central organization, with support from professional societies and networks (e.g., AAU, APLU, ASA, SIAM).

Both can be effective, and both are necessary because of the breadth of change under discussion. Additionally, within a university, partnerships with other disciplines, at both research and pedagogical levels, are key.

The group also discussed a precedent in the biological community, which had spent a decade transforming undergraduate biology education. Over the previous several decades, the content and pedagogy of the standard introductory biology courses had become stale and inconsistent with modern research. For this reason, the community came together to recommend which core concepts biology students should master. Significant NSF support led to the creation of new course materials and updated pedagogies, including the use of teaching fellows who helped spread the transformation. The community also created a framework for institutional change through rubrics and standards to assess a department's readiness for change, and regional networks supported individual departments through the process.

Recommendations for general learning

1. **Productive persistence:** Ability to work through and learn from mistakes and tolerate short-term failure; to stay engaged despite setbacks; and to stay on task while keeping sight of overall goals.
2. **Deep understanding:** To grasp the essential features of mathematics being studied, beyond the rote or algorithmic aspects.
3. **Problem solving:** Ability to bring relevant mathematics to bear on a problem, without knowing in advance what sort of mathematics is needed.
4. **Communication:** Capacity to convey mathematical ideas at a level appropriate to the needs and background of the audience.

With few exceptions, the nation's colleges and universities do not evaluate the extent to which pedagogy is effective in producing these desired outcomes. For example, if the ability to communicate mathematics is a desired goal, we must provide students with opportunities to communicate mathematics to others.

Recommended changes in the classroom

1. Increase instruction in use of data, statistics, modeling, and computation. Because this is in line with the Common Core Math Standards, it is particularly important in training future teachers.
2. Develop experiential learning and internship opportunities through partnerships with business, industry and government.
3. Ensure the effective use of technology in the classroom.
4. Provide ample advising and access to career services.

14 | Recommended changes in teacher support

1. **Colleague mentoring:** Faculty leaders can help disseminate new teaching methods by recruiting and mentoring existing faculty. Incentives such as release time may reward the time and effort spent *outside* the classroom to acquire new skills, new language, and new colleagues.
2. **Learning assistants:** Teaching can be augmented by appropriately trained and supervised undergraduate learning assistants.
3. **Disseminate best practices:** Create or raise the profile of structures (conferences, online communities, perhaps via professional societies) to disseminate best practices among faculty and departments.

Recommended means of implementation

It is difficult for individual mathematical science departments to generate widespread change. This is due, in part, to the large number of stakeholders in teaching and learning mathematics: other departments on campus whose students take mathematics courses; college deans and administrators who allocate budgets across departments; individual faculty within and outside mathematics departments; future employers of students; and the students themselves.

If any stakeholders view the math department as the only agent promoting change, the chances of improvement are diminished. Rather than math department chairs conducting individual bilateral discussions with single stakeholders, they must engage in simultaneous, multi-lateral discussion of desirable steps, such as:

1. TPSE Math may serve as a facilitator and “lubricator” among large university organizations, beginning with AAU, APLU, and AMATYC; professional societies, such as AAAS, AMS, MAA, ASA, SIAM, and JPBM; state educational agencies; federal agencies, such as NSF, PCAST, and the federal 5-tier STEM plan; and ongoing projects (see Appendix 2).
2. Adopt and disseminate rubrics and methods to help departments identify and evaluate opportunities for improvement. Create regional partnerships to foster and support transformation.
3. Create a repository for course materials and information about course and program transformation. Ensure the sustainability of the archive.
4. Promote widespread use of data, evidence, and assessment.

Individual contributions

One group member proposed a concrete way to begin. He suggested raising several million dollars to support a peer mentoring program at several dozen institutions. Those already working on these issues would recruit others who are willing, but lack capability to get involved. Such mentoring could serve as a stimulus to administrators to contribute resources.

Multiple participants pointed out that most of the changes recommended must occur at the departmental level, although not in isolation from the rest of the stakeholders. This means that the agents of change are the faculty, who must be properly supported and rewarded. A group member said that in his experience, “teaching awards had an interesting effect.” He recommended that individuals who make contributions to reform be rewarded by professional certification or recognition, perhaps as “TPSE Math Fellows.”

A third participant noted the success of a “professors-in-residence” track that rewards faculty who have visibility in institutions where pedagogy receives special emphasis.

Several participants at both the JMM and Austin meetings noted that desirable change in undergraduate biology teaching had been hastened by the use of updated textbooks.

Group 5 / Graduate Training

The discussion of the group on graduate education took as its starting point the fact that the post-degree world has changed dramatically in the recent past, and probably will continue to do so.

General finding

Despite the fact that some 60% of all mathematics PhDs will find employment outside the traditional Group I-III departments—therefore doing more teaching than research—most graduate training does not equip them for such careers.

General recommendation

In light of the diverse careers open to math PhDs, programs should promote the development of teaching and mentoring skills; communication skills, especially with those outside the profession; and internships in the private sector. In addition, graduate candidates should be familiar with the merits of masters programs, which are often designed to meet employment needs or serve as feeder programs to PhD programs.

Specific findings

1. According to recent data from the AMS, about 60% of all mathematics PhDs will eventually find employment outside the traditional Group I-III mathematics departments (that is, the kinds of department from which they graduated). This is an opportune time to adjust elements of graduate training to better prepare future graduates for this changing world.
2. For PhD recipients in mathematics who do not end up in Group I-III research departments, a significant fraction go to academic institutions that emphasize undergraduate education, or to industry or government laboratories. The group considered this to be a positive state of affairs, and an essential means of transferring vital mathematical expertise to other domains. At the same time, traditional graduate education has done little to prepare them for the wide breadth of careers now available.
3. While a traditional goal of graduate programs in mathematics is to develop excellent early-career mathematicians, many departments now emphasize masters programs, demand for which is significant and growing. Many such programs emphasize career advancement and professional development of relevance to many positions in government and the private sector, especially in fields of finance, bioinformatics, and cryptography. Masters content taught to middle school and high school teachers is also valuable in supporting development of mathematical content.
4. Changes underway in undergraduate programs must be accompanied by changes in graduate programs, so that current graduate students will be better equipped as teachers—especially given the reforms now being proposed. The section below contains suggestions for doing this.

16 | Recommended changes in teacher training

1. Allow all graduate students at some (advanced) stage of their training to be in charge of their own class (i.e., not just act as a recitation instructor). This may already be happening at many larger (state) institutions, but it is equally important in smaller (private) programs. This responsibility should be accompanied by appropriate training and pedagogical instruction, so that it both responds to the needs of undergraduate students and advances the teaching skills of the graduate student.
2. Promote the development of communication skills of graduate students, especially with people outside the profession. This could be done through student-sponsored seminars as well as through special department-sponsored events with outside “coaches.”
3. Strongly encourage networking activities at the graduate student level. As an essential element of early career development, networking options could be developed, for example, by an appropriate expansion of Project NeXT and by increased use of the Mathematics Research Communities program.

Recommended changes in curricula

1. Increase exposure to computational work that is relevant to the student’s area of research and, more generally, to a potential job or career. Such exposure could be facilitated by a wide-ranging course or seminar that would introduce such desirable skills as computation, statistics, and basic ideas of continuous and discrete modeling.
2. Increase emphasis on mentoring by graduate students of other graduate students and undergraduates, much in the spirit of a Research Training Grant.
3. Increase access to internships to familiarize graduate students with a range of employment opportunities, especially in the private sector. Because not all departments have the resources to do this, a national database of internship offerings should be created, possibly by one or more of the national societies or NSF-sponsored national institutes.

Recommended guidelines for masters programs

1. In the interest of diversifying pathways to mathematically intensive careers and better build a mathematically educated workforce, continue to design and implement more mathematics masters programs. In particular, this degree may help address the need for better-trained teachers at secondary and post-secondary levels.
2. Implement more masters programs that serve as feeders to PhD programs for undergraduates with weaker backgrounds without lowering the level of those PhD programs.

Next Steps

In addition to the topics of the breakout groups, several issues were emphasized by multiple participants and might be taken to indicate themes of wide interest:

Diversity outreach efforts

Participants at the meeting heard about and discussed efforts to increase the number of undergraduate and graduate students from traditionally underrepresented groups, including female, African-American, and Latino students. While there was strong support for these individual efforts, there was also a

feeling that such efforts should be sustained by national societies or other large organizations if they are to develop long-term effectiveness. Critical mass is important, and both alumni and current graduate students can be tapped as part of an effective recruiting effort.

Several attendees broadened the conversation to call for the participation of more faculty and students from smaller institutions, including HBCUs, where education and training are often of higher priority than research. Accordingly, TPSE Math is planning a more representative membership and more extensive institutional partnerships.

Scaling

TPSE Math is well aware that many individuals, departments, and institutions have already taken leadership roles in transforming post-secondary mathematics education, especially in diversifying tracks for students in bio-medicine, business, and engineering. It is also aware of the lack of mechanisms to improve on these innovations or, when warranted, to scale early experiments to institutional, regional and even national scale. Indeed, this is virtually impossible for individual departments, where many of the most innovative ideas may be found.

Scaling in higher education is not merely replication of a particular experiment because of the enormous diversity among institutions—even those in the same Carnegie classification. Scaling must be done with integrity rather than strict fidelity, so that new ideas are blended with local particulars.

Even the most effective projects may not scale easily. One reason is that they are not designed with scaling in mind. A second is the lack of understanding of many good mathematicians about the policies which can inhibit the spread of great ideas that actually meet an externally validated evidence bar. Education problems carry a certain dimensionality, and solutions of too-low dimension tend to fail.

To surmount such difficulties, TPSE Math proposes networking with existing large structures, notably the AAU, APLU, and AMATYC, which represent the majority of research universities, land-grant universities, and community colleges, respectively. TPSE Math proposes that one must use national-scale organizations to generate national innovations. It hopes to “surface” and share promising local practices, encouraging the mathematics community to design the details of the scaling.

Tracking and involving alumni

The discussion in Austin of diversity issues in mathematics overlapped with a related discussion of the role of alumni in influencing the education and training of students. Far too often alumni with non-academic employment are not tracked by departments and contact with them is lost. This deprives departments of potential mentors, valuable supporters, donors, employers, sponsors of interns, and even ambassadors to society. Alumni from underrepresented groups may be especially effective at introducing young members of those groups to appropriate pathways through a mathematics career. Therefore, participants recommended that departments should view tracking of alumni (in particular those with non-traditional employment) as an important task to be supported and encouraged by the mathematics community.

A future role for TPSE Math

The TPSE Math group that organized this meeting has discussed a continuation of its activities as an “enabling body.” It anticipates activities that include a combination of outreach, networking, information gathering, publicity, and support for mathematicians already developing and evaluating new models and techniques of education.

It also sees value in the role of brokering partnerships across the often-fragmented mathematics community—causing good things to happen by suggesting appropriate partnerships. The group hopes to build on innovative ideas already being implemented at local levels, rather than formulating new ideas that are likely to be duplicative. TPSE Math believes that a well-played brokering role can promote positive culture change by cross-fertilizing “around the edges” and promoting a collective sense of where things need to move.

The strong showing, enthusiasm, and productive discussions in Austin have encouraged the TPSE Math group to plan for additional regional meetings at diverse institutions, including community colleges, R1 universities, and liberal arts colleges. Among general takeaways from Austin are a stronger awareness of the many innovations already in process and the needs for scaling up the most successful of these.

We have also become aware of several important trends and forces:

- Departments contemplating curriculum changes for the first two years of post-secondary education are often constrained by budgetary pressures and the demands of client departments;
- In some cases, it may be that 2-year and 4-year colleges have more independence and opportunity to initiate reforms than large universities;
- Client disciplines are sometimes consulted by mathematics departments as partners, some of them seeking more focus on math skills and others more focus on applications;
- Pedagogical innovation is typically driven by one or a few committed faculty members, but their new techniques may fare poorly when adopted by others;
- Non-exceptional students tend to benefit from personalized education that offers small class size and mentoring.

Successful continuation of this project will require more knowledge about such trends, which can be gathered through additional networking, personal communications, expansion of the TPSE Math website, and enhanced outreach and public education.

The TPSE Math group has begun to plan additional meetings and to seek additional funding to carry proposed reforms beyond the current exploratory stage.

Endnotes

1. President’s Council of Advisors on Science and Technology, *Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics*, Executive Office of the President, February 2012.
2. National Research Council, *Moving Beyond Myths: Revitalizing Undergraduate Mathematics*, National Academy Press, Washington DC, 1991.
3. National Research Council, *The Mathematical Sciences in 2025*, The National Academies Press, Washington, DC, 2013.
4. Current members of TPSE Math are Phillip A. Griffiths, Institute for Advanced Study, Convener; Eric Friedlander, University of Southern California; S. James Gates, Jr., University of Maryland; Mark Green, UCLA; Tara Holm, Cornell University; and Uri Treisman, University of Texas at Austin.
5. http://www.tpsemath.org/jmm_panel_summary
6. See Engage to Excel, pp. 27-30; also, University of Maryland study.

Agenda

Friday, June 20, 2014

- 2:00–2:15 Welcome and introduction: *Uri Treisman*, UT Austin
- 2:15–3:00 Keynote Address: *Bill Powers*, President, UT Austin
- 3:00–3:15 Guidelines to working groups: *Phillip Griffiths*, IAS
- 3:15–5:15 Primary working groups begin review of key questions:
- Curriculum reform
 - Opening pathways
 - Technology, teaching, and economic impacts
 - Enriching and broadening the undergraduate experience
 - Enriching and broadening graduate training

Saturday, June 21, 2014

- 8:30–9:00 Challenges/Values/Outcomes: *Eric Friedlander*, USC
- 9:00–10:30 Primary working groups continue
- 11:00–12:30 Primary working groups continue; develop preliminary conclusions.
- 2:00–3:30 Secondary working groups convene (chairs, note-takers & TPSE representatives remain in primary groups)
- 4:00–5:30 Plenary session: presentations by group leaders of preliminary conclusions; open discussion. Moderator: *Phillip Griffiths*

Sunday, June 22, 2014

- 9:00–9:30 Overview of what has been accomplished so far; guidelines to working groups for final breakout sessions: *Phillip Griffiths*
- 9:30–9:45 Observations and Perspectives: *Joan Ferrini-Mundy*, NSF
- 9:45–10:45 Primary working groups reassemble to consider all feedback and consolidate conclusions, recommendations and actions.
- 11:00–12:30 Panel of working group chairs presents conclusions, recommendations, actions; open discussion. Moderator: *Phillip Griffiths*
- 12:30–12:45 Concluding remarks: *Greg Fenves*, Provost, UT Austin

Meeting Hosts

William Powers, President, University of Texas at Austin

Greg Fenves, Provost, University of Texas at Austin

TPSE Math Leadership Team

Eric Friedlander, University of Southern California

S. James Gates, University of Maryland

Mark Green, University of California—Los Angeles

Phillip Griffiths, Institute for Advanced Study

Tara Holm, Cornell University

Uri Treisman, University of Texas at Austin

Participants

Malcolm Adams, University of Georgia

Matthew Ando, University of Illinois at Urbana-Champaign

Jayadev Athreya, University of Illinois at Urbana-Champaign

Larry Bacow, Tufts University

Matthias Beck, San Francisco State University

William Beckner, University of Texas at Austin

David Bressoud, Macalester College

Ed Burger, Southwestern University

Jason Cantarella, University of Georgia

Herb Clemens, University of Utah; Conference Board of the Mathematical Sciences

James Curry, National Science Foundation

Mark Daniels, University of Texas at Austin

Stephen DeBacker, University of Michigan

Jennifer Dorsey, University of Texas at Austin

Joan Ferrini-Mundy, National Science Foundation

Bernadine Fong, Carnegie Foundation for the Advancement of Teaching

Robin Forman, Emory University

Melissa Goessling, University of Texas at Austin

Daniel Goroff, Alfred P. Sloan Foundation

Rebecca Griffiths, Ithaca

Linda Hicke, University of Texas at Austin

William (Bus) Jaco, Oklahoma State University

Harrison Keller, University of Texas at Austin

William (Brit) Kirwan, University System of Maryland

Phil Kutzko, University of Iowa

Joan Leitzel, Ohio State University

David Levermore, Society for Industrial and Applied Mathematics (SIAM)

Jim Lewis, University of Nebraska

Harry Lucas, Education Advancement Foundation

Peter March, Ohio State University

Gary Martin, Auburn University
Don McClure, American Mathematical Society (AMS)
Jennifer McNeilly, University of Illinois at Urbana-Champaign
Alison Miller, University of Texas at Austin
Haynes Miller, Massachusetts Institute of Technology
David Morrison, University of California—Santa Barbara
Rachel Mudge, Foothill College; Carnegie Foundation for the Advancement of Teaching
Mary Nelson, George Mason University
Bruce Palka, University of Texas at Austin
Jennifer Pearl, National Science Foundation
Michael Pearson, Mathematical Association of America (MAA)
Jill Pipher, Brown University
Alan Reid, University of Texas at Austin
Danielle Rutherford, University of Texas at Austin
Nancy Sattler, Terra State Community College; American Mathematical Association of Two-Year Colleges (AMATYC)
Frank Savina, University of Texas at Austin
Padmanabhan (Padhu) Seshaiyer, George Mason University
Martha Siegel, Towson University
Susan Singer, National Science Foundation
Richard Smith, University of North Carolina—Chapel Hill; Statistical and Applied Mathematical Sciences Institute (SAMSI)
Dalene Stangl, Duke University
Michael Starbird, University of Texas at Austin
Francis Su, Harvey Mudd College
Jeremy Teitelbaum, University of Connecticut
Maria Terrell, Cornell University
Peter Turner, Society for Industrial and Applied Mathematics (SIAM)
Michael Vogelius, National Science Foundation
Judy Walker, University of Nebraska
Paul Zorn, St. Olaf College; Investing in the Next Generation through Innovative and Outstanding Strategies (INGenIOuS)

Staff

Alan Anderson, writer/editor
Arlen Hastings, TPSE Math administrator, Institute for Advanced Study
Rachele Seifert, UT Austin
Lilly Soto, UT Austin

Note Takers

Jen Dorsey, UT Austin
Alison Miller, UT Austin
Danielle Rutherford, UT Austin
Frank Savina, UT Austin

Breakout Groups

A. Curriculum Reform

Group Chair: Leitzel

TPSE Representative: Green

Note Taker: Dorsey

Primary Group Members: Daniels, DeBacker, Jaco, Levermore, Lucas, H Miller, Pearson, Pipher, Siegel, Stangl, Terrell, Turner, Zorn

Secondary Group Members: Ando, Beckner, Bressoud, Fong, Massey, Morrison, Mudge

B. Opening Pathways

Group Chairs: Treisman, Gates

TPSE Representatives: Treisman, Gates

Note Taker: Savina

Primary Group Members: Adams, Athreya, Fong, Kutzko, Lewis, McNeilly, Mudge, Nelson, Su

Secondary Group Members: Beck, Clemens, Curry, Goroff, R Griffiths, Jaco, Keller, McClure, Reid, Sattler, Seshaiyer, Smith, Terrell, Turner

C. Technology/Teaching/Economics

Group Chair: Bacow/Kirwan

TPSE Representative: P Griffiths

Note Taker: Rutherford

Primary Group Members: Beckner, Cantarella, Forman, Goroff, R Griffiths, Keller, Morrison, Sattler, Teitelbaum

Secondary Group Members: Levermore, H Miller, Nelson, Pipher, Starbird, Su, Walker

D. Undergraduate Experience

Group Chair: March

TPSE Representative: Holm

Note Taker: Miller

Primary Group Members: Ando, Bressoud, Clemens, Martin, Reid, Seshaiyer, Singer, Starbird

Secondary Group Members: Adams, Cantarella, Daniels, Debacker, Forman, Lewis, Lucas, McNeilly, Palka, Pearl, Pearson, Siegel, Singer, Stangl, Teitelbaum, Zorn

E. Graduate Training

Group Chair: Vogelius

TPSE Representative: Friedlander

Primary Group Members: Beck, Curry, Massey, McClure, Palka, Pearl, Smith, Walker

Secondary Group Members: Athreya, Kutzko, Martin

Some Previous and Existing Efforts at Transformation

The mathematical sciences community is home to a great deal of thoughtful and ongoing activity in the areas of teaching and learning and curriculum development. Many of these have originated in individual departments, such as:

- A first-semester online calculus for matriculated students who would otherwise have difficulty enrolling (University of Washington, Seattle);
- A capstone course along with the foundations courses (University of Texas at Austin);
- A freshman course with a major MatLab component (Northeastern University).

In addition, AMATYC, SIAM, ASA, and MAA have been (and are currently) involved in formulating curriculum recommendations for undergraduate mathematical science students. All of the major professional organizations have active committees on education. The curriculum development activities of the professional organizations have made a practice of taking into account the needs of other mathematically intensive disciplines. The AMS provides awards to departments that excel in their undergraduate mathematics programs, bringing examples of excellence to the community. The Inquiry Based Learning movement has established a vibrant community, with several centers around the country, an annual meeting, and a group of textbook authors who help enable more people to try this method of teaching. Other efforts have involved the production and use of MOOCs, web-based homework systems, etc. Professional organizations have endorsed undergraduate research with awards, poster sessions, and talks at regional and national meetings. Many of the projects presented at meetings are interdisciplinary. Pi Mu Epsilon and a number of collegiate research journals routinely publish mathematical research papers by undergraduates. Math Horizons is a magazine specifically for undergraduates interested in mathematics. AMS's Mathematical Moments are excellent resources for the classroom. Endeavors such as Math Awareness Month, COMAP, the use of undergraduate Learning Aides, and undergraduate student chapters of the professional organizations have added to the richness both in and outside of the academy.

We include a sampling of the larger national efforts (such as conferences, workshops, reports and projects), and a small fraction of the very many local efforts at implementation:

Modeling across the Curriculum

There have been two "Modeling across the Curriculum" workshops resulting from NSF funding to SIAM, with PI's Jim Crowley (SIAM) and Peter Turner (Clarkson University). The first workshop, held at NSF in August 2012, had three themes: Modeling in the Undergraduate Curriculum, Supporting Modeling Courses in K-12 (especially high schools), and Assessment and Evaluation. The major outcomes were summarized in the report, available at www.siam.org/reports/modeling_12.pdf. Arising from the high school recommendations and the Moody's Mega Math Challenge, a handbook on modeling for both students and coaches was also produced: *Math Modeling: Getting Started and Getting Solutions*, by Bliss, Fowler and Galluzzo.

The second workshop, MaC II, was held at the ASA in January 2014. The themes were again centered on undergraduate and K-12 curricula. Following the discussions of MaC I, the issue of whether it is better to adopt stand-alone courses or an "infusion" approach continued. Discussions focused on programs and courses, materials, and professional development and training. The report on MaC II is anticipated for publication later in 2014.

Common Vision for Undergrad Math in 2025

This is a new initiative led by MAA (PI is Karen Saxe, Macalester College) in collaboration with AMATYC, AMS, ASA, and SIAM. The plan is for a workshop in early May 2015. The intention is to look at undergraduate math education, especially in the first two years, with a very broad range of targets including math and other STEM majors, non-STEM majors, quantitatively informed citizens, the growth in data-based science, logical and analytic thinking, etc. In this broad context, how do we design programs, courses and pedagogies, and how do we use technologies to help increase student success? What support will be necessary for students, for faculty and for future teachers? And, how will we know if it has worked? How will approaches need to vary across different programs and institutions? The intention is to address problems related to recruitment and retention of STEM students, especially of women and minorities, sustaining a mathematically literate population, forging links to STEM and non-STEM fields, and advancing timely completion rates.

Undergraduate Programs in Applied Mathematics

SIAM has recently published its report on undergraduate applied mathematics programs. This is not restricted to majors but includes guidance on minors and other smaller programs of study. It is available at www.siam.org/

reports/undergraduate_14.pdf. With a global focus on preparing students to enter STEM fields, undergraduate programs in Applied Mathematics have an important role to play in preparing the future workforce. The purpose of this advisory report is to describe the components of existing programs in Applied Mathematics. The intended audience includes people who may be interested in initiating new programs, improving existing programs, or policy-makers. It is not intended to be prescriptive but rather to describe some of the approaches, with suggestions for content. Tabular data includes courses that are almost always present in applied programs and others that are commonly included.

The INGenIOuS Project

The INGenIOuS Project is a workforce development effort of four professional societies—the AMS, ASA, MAA, and SIAM—with support from the NSF. Project activities have included online panels, commissioned white papers on various workforce-related themes, and a July 2013 workshop, with participants representing academia, institutes, business, industry, government, and foundations. Among key project recommendations are (i) the need to attract, train, and retain an increasingly diverse cohort of students and professionals to mathematics- and statistics-rich careers ; and (ii) the need to develop curricula and academic reward structures that lead to better student preparation for careers both outside of and inside academia.

Much more information can be found at www.ingeniousmathstat.org. Materials there include sample project descriptions with possible evaluation metrics, white papers, background readings, and other resources.

Mathematical Association of America Committee on the Undergraduate Program in Mathematics

MAA will soon release the *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences*, developed with help from over 200 mathematicians and colleagues in STEM disciplines. The *Guide* offers a framework for all undergraduate majors in the mathematical sciences, regardless of track, emphasis, or interdisciplinary area. Each department is urged to carefully and intentionally consider its distinctive mission, students, and resources in designing a curriculum that helps its students mature mathematically and gain essential skills. Sections of the *Guide* also address non-curricular areas—placement, recruitment and retention, articulation, and technology—that support and inform curriculum. The MAA intends the *Guide* to become a dynamic online resource, drawing on the expertise and interest of a growing community dedicated to the improvement of collegiate mathematics.

Since 1953, MAA CUPM has issued reports and recommendations on the full range of the undergraduate mathematics curriculum. CUPM subcommittees address such particular areas as the first two college years, undergraduate research, and mathematics across the disciplines. The *2015 CUPM Guide* incorporates suggestions from review groups representing the American Mathematical Society, the Association for Women in Mathematics, the American Statistical Association, the Society for Industrial and Applied Mathematics, the American Mathematical Association for Two-Year Colleges, and other members of the Conference Board of the Mathematical Sciences. For additional CUPM history and an archive of reports, see maa.org/cupm.

The Calculus Study (CSPCC)

In 2009 MAA began an extensive study of Calculus I, involving a national survey of departments, instructors, and students undertaken in 2010, followed by case study visits in 2012 to 17 colleges and universities engaged in promising practices. The intent has been to better understand who takes calculus, why they take it, and what happens to their attitudes toward mathematics and intention to persist in a STEM career as a result of this course; to better understand how it is taught and how the choice of pedagogical strategies affects student outcomes; and to be able to make recommendations to the mathematical community on policies and practices that improve the teaching and learning of calculus. Further descriptions as well as links to results can be found at maa.org/cspcc.

MAA Special Interest Group on Research in Undergraduate Mathematics Education (SIGMAA on RUME)

The RUME community has recent and ongoing NSF projects in the learning and teaching of (among others) proof, abstract algebra, multivariable calculus, differential equations and linear algebra. A new, soon-to-be-published *Handbook on Research in Mathematics Education* includes chapters summarizing research on the teaching and learning of multiple undergraduate topics. It also has collaborations with people who work in undergraduate physics and chemistry education. See sigmaa.maa.org/rume/Site/News.html.

ISSUES Workshop

The initial meeting of the project *Integration of Strategies that Support Undergraduate Education in STEM* (ISSUES) was held in January 2014 at the MAA Carriage House with representatives from disciplinary societies in the biological, mathematical, and physical sciences and engineering, as well as representatives of AAU, APLU,

AAC&U, AAAS, HHMI, and NSF. The purpose was to find areas in which disciplinary societies could learn from each other's activities, coordinate efforts and policy statements to increase their collective effectiveness, and leverage their influence to help build interdisciplinary connections directed toward improving undergraduate education within individual colleges and universities. The website for this effort is at serc.carleton.edu/issues. An explanation of specific goals for this project can be found at launchings.blogspot.com/2014/03/collective-action-by-stem-disciplinary.html

Ohio Postsecondary Mathematics Initiative

The Ohio Board of Regents is the coordinating board for public higher education in Ohio. In the summer of 2013, OBR formed a Mathematics Steering Committee to review emerging issues in postsecondary mathematics education and to make recommendations for changes that were seen as needed. The full report and an executive summary can be accessed at ohiohighered.org/mathematics-initiative-documents. The chairpersons of the mathematics departments at Ohio's 36 public institutions have undertaken the implementation phase of this work and are now meeting together and moving in a coordinated manner to establish needed mechanisms for change in all departments.

Ohio State University MOOCs

The Mathematics Department at Ohio State undertook in 2012 to develop MOOCs in calculus. Jim Fowler has led the work with a team of mathematicians, math educators, and technical staff. In 2013 MOOCs were offered in both Calculus I and Calculus II. Course materials and other information are available at mooculus.osu.edu.

Research Experiences for Undergraduates

This is a longstanding and highly successful program, funded by the NSF, that offers research experiences in the mathematical sciences for undergraduates. There are 49 sites currently listed on the NSF website: nsf.gov/crssprgm/reu/list_result.jsp?unitid=5044.

NSF Mathematical Sciences Institutes

The NSF Research Institutes in the Mathematical Sciences offer a wide variety of programs devoted to undergraduate education. The institutes constitute an important resource for bridging between emerging research directions and the undergraduate experience. A portal to all of the institutes is at mathinstitutes.org.

The Discovery Learning Project

The Discovery Learning Project (www.discovery.utexas.edu/dlp) promotes the development and use

of discovery- or inquiry-based methods of teaching and learning. This project is predicated on the belief that all students have creative ability which can be further developed utilizing the many techniques of inquiry-based learning. The University of Texas at Austin has an extensive history of innovation in this style of teaching and learning. UT professors R. L. Moore in mathematics and R. N. Little in physics each developed inquiry-based approaches to teaching that seek to transform students from "knowledge consumers" to "knowledge creators and interpreters." The Discovery Learning Project builds on that legacy of innovation by developing inquiry-based learning opportunities in line with recommendations of the National Science Foundation report *Shaping the Future: New Expectations for Undergraduate Education in Science, Mathematics, Engineering, and Technology* (www.nsf.gov/publications/pub_summ.jsp?ods_key=nsf96139) and of the National Research Council report *From Analysis to Action: Undergraduate Education in Science, Mathematics, Engineering, and Technology* (www.nap.edu/catalog.php?record_id=9128). The goal is to improve students' abilities as independent thinkers and to encourage them to become life-long learners. The project is supported by the Educational Advancement Foundation (eduadvance.org) and the College of Natural Sciences.

The Academy of Inquiry-Based Learning:

AIBL (inquirybasedlearning.org) is a community of instructors, teachers, and supporters of inquiry-based learning whose philosophy is "big tent IBL." AIBL takes a broad view of inquiry-based learning, supporting the use of a wide range of teaching methods in mathematics courses consistent with courses where students are (a) deeply engaged in rich mathematical tasks, and (b) have ample opportunities to collaborate with peers (where collaboration is defined broadly).

One of the goals of AIBL is to become a national resource for IBL methods for a wide range of courses and institutional environments. While AIBL supports instructors interested in teaching a full IBL course (or Modified Moore Method), it also support those interested in modifying and adapting IBL to suit specific, local needs.

Journal of Inquiry-Based Learning in Mathematics:

The JIBLM (jiblm.org) publishes university-level course notes that are freely downloadable, professionally refereed, and classroom-tested. These course notes are intended for instructors to use as a starting point to design a course that meets the specific needs of their students, and for students to use to study a topic inde-

pendently. Instructors are encouraged to contact authors for guidance in the use of the course notes on this site.

Center for Inquiry Based Learning at the University of Michigan

The University of Michigan hosts one of four national IBL Centers (math.lsa.umich.edu/ibl), along with the others at the University of Texas at Austin, the University of California at Santa Barbara, and the University of Chicago. The Michigan Center promotes the teaching and development of inquiry-based learning (IBL), a method that emphasizes discovery, analysis, and investigation to deepen students' understanding of the material and its applications. The Center has developed 13 full-scale IBL courses over the last ten 10 years, with more planned. These range from courses for freshmen to courses for math majors. In fact, all content courses in math education are now taught as IBL courses. In addition, several courses have IBL modules, which are units used once a week in a class to complement the lecture part of the course. The Center plays an important role in training faculty, postdocs, and graduate and undergraduate students to teach using techniques of IBL. In addition, it conducts various outreach activities including IBL workshops, IBL lunches, local workshops, a math circle for start-up teachers, and visits to other campuses to discuss IBL activities.

CBMS Forum

CBMS fora have focused on the preparation of teachers for K-12 especially in light of the development and implementation of the Common Core State Standards. A forum was held in October 2014 on *The First Two Years of College Math: Building Student Success* (www.cbmsweb.org/Forum5/index.htm). Three fundamental areas of interest are the pathways to different careers: in STEM, in life and social sciences, and through the two-year colleges and general education programs. In all cases, there is an intended emphasis on a mathematics education that is based on relevance to the real world and the breadth of potential careers. This likely implies a demand for a modeling and applications oriented curriculum that pays attention to appropriate conceptual understanding combined with relevant skills.

SIAM-EESI Workshop on Future of CSE Education

This small workshop was held in August 2014 with its primary focus on the future of Computational Science and Engineering (CSE or CS&E) and Data-Enabled Science and Engineering (DESE) at the graduate level, with some attention to undergraduate curriculum issues in this growing area of applications-oriented mathematical science. DESE can be viewed as a major expansion

of CSE, more explicitly incorporating big data and data-based models. The primary focus of both CSE and DESE is the multidisciplinary approach to advancing the science and engineering knowledge base through close collaboration with the mathematical sciences community. As such it can be viewed as explicitly encouraging a mathematics education of more relevance in the real world.

The Charles A. Dana Center

The Dana Center (www.utdanacenter.org) is an organized research unit based in the mathematics department at the University of Texas at Austin. Led by founding director Uri Treisman, the Center works on improving math education at scale in states and large urban districts. Its three signature programs in higher education are the New Mathways Project (NMP), the State Math Task Force Project, and the Emerging Scholars Program (ESP). NMP is a 10-year partnership with all of Texas' community colleges to modernize their math programming and to link their gateway math courses more directly to programs of study. The rapidly growing Math Task Force project is supporting senior leadership teams of mathematicians in seven states to shape and implement policies that improve state math education at scale. ESP, which in various forms is now widespread in public higher education, is designed to diversify the population of students pursuing math-intensive majors and professions. At the K-12 level, the Dana Center and its partners, the Aspen Institute and Agile Mind, provide technical support to many of the largest school districts in the U.S. which are working to improve their mathematics and science instructional programs. The work of the Dana Center has led to the creation of many successful curricula aimed at broadening the STEM base. One example is the MIST program at UIUC (merit.illinois.edu/index.html).

The Introductory Program at the University of Michigan

Michigan introductory classes—Pre-calculus (math.lsa.umich.edu/courses/105), Calculus I (<http://math.lsa.umich.edu/courses/115>), and Calculus II (math.lsa.umich.edu/courses/116)—are run in small interactive sections that focus on problem solving and conceptual understanding rather than solely computational literacy. Students are expected to actively participate in class—solving problems, working in groups, presenting solutions on the board, and critiquing the solutions of their peers. The structure of the introductory courses is critical to their success. In the spirit of the “flipped classroom,” students read the text before class so that more class time is devoted to actually doing and discussing

mathematics. Instructors spend less time lecturing and more time engaging with the students and are therefore better able to catch misunderstandings in real time. Students complete individual online homework assignments outside the classrooms, work on challenging team homework problems, and produce written reports on their work, explaining their reasoning and analyzing their conclusions. Through this process, students work with peers from many different backgrounds and disciplines, tackle difficult problems related to a variety of applications, and must learn to clearly communicate their logical reasoning. Exams require skill in working with graphs, tables, verbal descriptions, and formulas, and locally designed computer-based “Gateway Exams” ensure that students acquire computational skills. As measured by the Calculus Concept Inventory, the average performance of Michigan calculus students is reported to be more than two standard deviations above the national average (ams.org/notices/201308/rnoti-p1018.pdf).

Examples of Special Courses

contributed by Dan Freed, MIT

I favor the introduction of new courses and modes of teaching at both the undergraduate and graduate levels. Undergraduate mathematics majors, even those planning to go to graduate school, should be required to take not only the usual undergraduate courses in algebra and analysis but also to learn some statistics and numerical analysis, or two similar topics. Those classes should be designed so that students experience first-hand the uses of mathematics in other fields.

I also favor creating a few special courses which are not systematic training in specific subjects, but which educate more broadly about mathematics and being a mathematician. These should convey a broad conception of “mathematician” and “mathematics.” I offer two specific local examples that can be implemented more widely. The first is an undergraduate course on Mathematical Thought, contemplated for a new honors program, which has never been taught. The second is a graduate course we’ve run several times, which is modeled after Dan Kan’s long-running seminar at MIT.

Mathematical Thought: This course has two distinct goals. First, the students are expected to grasp basic concepts and tools (basic logic and proof, problem solving skills, the language of mathematics). They should also learn practical tools (not through lectures), including experience with computer-assisted computation and visualization, TeX, and basic knowledge of online

resources for mathematics. Students should also read essays and engage in discussions about mathematics, the study of mathematics, and mathematical careers. The second goal of the course is to communicate the excitement and vitality of mathematics. Students should experience—at the level of an excellent public lecture, say—some big achievements of mathematics in the past 50 years and also some big open problems. In some cases there may be lectures available online, which students can watch on their own and engage in class discussion and/or work problems around the lecture. In other cases members of our department can give one-off lectures to the class. The Mathematical Thought class will, via its structure, transmit a unified view of mathematics which does not erect walls between its various subfields.

Kan Seminar: The transition from book and classroom learning to research is the most difficult and intimidating for a graduate student. The Kan Seminar is designed to help students with this transition, and to serve as an apprenticeship in communication. The faculty member guides eight students in the skills necessary to read research mathematics and give seminar talks. Each student studies and presents three classic papers; there are no lectures by the faculty member, who instead meets with students individually throughout the semester. The papers grow in complexity and scope through the semester. Whereas the MIT version focuses on algebraic topology and homotopy theory, at UT students engage with the entire spectrum of geometry and topology. But the subject is not the main point; the format can be used with any subject. I can envision a variation which is particularly suited to initiating students into applied areas. As the semester progresses, a major challenge for students is to extract a mathematical “story” from their papers and to then express the authors’ ideas and vision in their own voice. This prepares students to formulate research problems based on the papers they read. Our experience with this class has been very positive. Through their shared experience, students gain great confidence and form a tight and collaborative cohort.

Acronyms

AAAS - American Association for the Advancement of Science

AAC&U - Association of American Colleges and Universities

AAU - Association of American Universities

AIBL - Academy of Inquiry Based Learning

AMATYC - American Mathematical Association of Two-Year Colleges

AMS - American Mathematical Society

APLU - Association of Public and Land-grant Universities

ASA - American Statistical Association

CBMS - Conference Board of the Mathematical Sciences

COMAP - Consortium for Mathematics and Its Applications

CSE - Computational Science and Engineering

CSPCC - Characteristics of Successful Programs in College Calculus

CUPM - Committee on the Undergraduate Program in Mathematics

DESE - Data-Enabled Science and Engineering

EESI - European Exascale Software Initiative

ESP - Emerging Scholars Program

HHMI - Howard Hughes Medical Institute

IBL - Inquiry-Based Learning

INGenIOuS - Investing in the Next Generation through Innovative and Outstanding Strategies

ISSUES - Integration of Strategies that Support Undergraduate Education in STEM

JIBLM - Journal of Inquiry-Based Learning in Mathematics

MAA - Mathematical Association of America

MIST - Merit Immersion for Students and Teachers

MOOC - Massive Open Online Course

NMP - New Mathways Project

NSF - National Science Foundation

OBR - Ohio Board of Regents

SIAM - Society for Industrial and Applied Mathematics

SIGMAA on RUME - Special Interest Group of the MAA on Research in Undergraduate Mathematics Education

STEM - Science, Technology, Engineering and Math



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